Performing Musical Structure: 
crux-\( \phi \) perceptions in Domenico Scarlatti’s sonata K. 380

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Proceedings of the fourth Conference on Interdisciplinary Musicology (CIM08)  

Background in analysis. In an earlier study (Harper 2007) the direct occurrence of the crux in \( \phi \) proportion was observed in a large percentage of two contrasting bodies of Scarlatti sonatas (Essercizi and Cantabile sonatas). In the Sonata in E Major, K. 380 - a sonata with distinctive folkloric characteristics - direct occurrence of the crux-\( \phi \) relationship is also found in exact mathematical proportion in both halves of the sonata. While it is not known if this proportion is deliberately conceived, Scarlatti’s structural construct is evidenced in and is the basis of this work.

Background in performance. Ten different recorded interpretations (Horowitz, Asperen, Smullyan, Browning, Pletnev, Coleman, Fadini, Li, Lipatti, Gilels) were chosen for comparison in K. 380 with performers on harpsichord, fortepiano, and piano. The digital audio editor Audacity 1.3.3 was used to study the performances. After repeats and extra audio materials being removed, timings ranged from the longest (Horowitz and Asperen) at 3'10''9 to the shortest (Gilels) at 2'31''5. Expressive elements, such as variations in dynamic contrasts and rubati, are clearly discernible. The crux was compared in the ten performances first in real time and then in equalized time using a time-based analytical approach.

Aims. This study aims to look at the interpretative treatment of the crux - a proportional structural point in both halves of a Scarlatti sonata - and to determine a) if there is a relationship between structure and expression at the crux point, and b) if so, how the artists chose to express it. Although it was not possible to interview the artists represented in this study, it was possible to view their respective approaches to the crux placement through technological applications.

Main Contribution. In our study of these ten performances, we found that there is a tendency for the occurrence of the crux to converge in absolute time, implying the existence of an underlying tempo-structure relationship. We distinguish, moreover, between two fundamental tempi: Surface Tempo and Background Tempo. The former encompasses the individual expressive and personal interpretation, while the latter relates to a deeper and more fundamental notion of time that aligns absolute time with musical structure.

Implications. The musicological interdisciplinary implications of this study are twofold: 1) analytical tools scientifically allow a close scrutiny of how the performer communicates structure in music, specifically crux placement in a Scarlatti sonata; 2) performers, whose responsibility is to re-create and discover the sense of music, provide analysts with interpretative material regarding musical structure.

Pursuant to an earlier study that revealed a high incidence of crux-\( \phi \) relationship in two contrasting bodies of Scarlatti sonatas, it was decided to further analyze performers’ approach to this phenomenon in order to find out what kind of interpretative treatment, if any, might be given. The Sonata in E Major, K. 380, was chosen because of its popularity in widely available recordings and because the crux-\( \phi \) placement is in direct mathematical proportion in both halves of the sonata. The crux interpretation was then analyzed in ten performances with the digital audio editor Audacity 1.3.3 and subsequently compared to interpretations that were time modified, i.e. time equalized to a reference length using the audio editor Peak 5 software in order to draw conclusions about interpretative treatment of structural material.

Characteristics of K. 380

K. 380, in E Major, is one of the most popular of Domenico Scarlatti’s sonatas. Only 78 bars long, its two halves divide into 40 and 38 bars respectively. It follows the typical Baroque bipartite scheme in that the first half ends in the Dominant while the second half returns to the Tonic. At first glance, it seems to be a
simple and straightforward structure. However, Scarlatti’s genius belies his simplicity, for the richness of material and the subtlety of its treatment can be viewed in various ways and is often ambiguous.

The second half of the sonata undergoes a remarkably advanced harmonic treatment that foreshadows Schubert in its richness and romantic expression of a cantabile contour, while reminding us that Scarlatti was also a virtuoso singer. The closing material of both halves is similar. The range of the sonata (B1-g’’) reveals that it was written for an instrument beyond the early Italian or Portuguese harpsichords. However, it is adequately suited to a 5-octave instrument. It could feasibly be composed for fortepiano, a point defended by Fadini in her recording on a fortepiano by Anton Walter of 1790. On the other hand, performs on a copy of an Antwerp 1745 Dulcken harpsichord, and is obliged to compensate the lack of range by taking portions of the second half of the sonata down an octave.

**Kirkpatrick, the Crux, and Phi**

Ralph Kirkpatrick, through his monumental work, observed a phenomenon in the Scarlatti sonatas which he termed “the crux”. He defined it as “the point in each half at which the thematic material, which is stated in parallel fashion at the ends of both halves, established the closing tonality.” Thus, the crux has a triple function: melodic, harmonic, and structural.

Following Kirkpatrick’s definition, the crux in K. 380 occurs in the first half in ms. 25 (musical example 4) and in the second half in ms. 63, which corresponds to ms. 23 (musical example 5). Taking the phi (φ) measurement of each half (40 ms. x .618034 = 24.72136 and 38 ms. x .618034 = 23.485292), the crux is therefore in direct correspondence to the phi proportion.
Performing K. 380 in real time

Ten different recorded performances of K. 380 were selected. Eight were performed on modern piano and one each on fortepiano and harpsichord respectively by Vladimir Horowitz, Mikhail Pletnev, Chase Coleman, Raymond Smullyan, Yundi Li, Dinu Lipatti, Emil Gilels, John Browning, Emilia Fadini and Bob van Asperen (Table 1).

<table>
<thead>
<tr>
<th>Performer</th>
<th>Instrument</th>
<th>Repetitions</th>
<th>Total Duration</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vladimir Horowitz</td>
<td>Piano</td>
<td>No</td>
<td>3'11&quot;</td>
</tr>
<tr>
<td>Bob van Asperen</td>
<td>Harpsichord</td>
<td>Yes</td>
<td>6'21&quot;</td>
</tr>
<tr>
<td>Raymond Smullyan</td>
<td>Piano</td>
<td>No</td>
<td>3'00&quot;</td>
</tr>
<tr>
<td>John Browning</td>
<td>Piano</td>
<td>No</td>
<td>3'11&quot;</td>
</tr>
<tr>
<td>Mikhail Pletnev</td>
<td>Piano</td>
<td>Yes</td>
<td>5'43&quot;</td>
</tr>
<tr>
<td>Chase Coleman</td>
<td>Piano</td>
<td>No</td>
<td>2'47&quot;</td>
</tr>
<tr>
<td>Emilia Fadini</td>
<td>Fortepiano</td>
<td>Yes</td>
<td>5'13&quot;</td>
</tr>
<tr>
<td>Yundi Li</td>
<td>Piano</td>
<td>Yes</td>
<td>5'13&quot;</td>
</tr>
<tr>
<td>Dinu Lipatti</td>
<td>Piano</td>
<td>No</td>
<td>2'33&quot;</td>
</tr>
<tr>
<td>Emil Gilels</td>
<td>Piano</td>
<td>Yes</td>
<td>5'04&quot;</td>
</tr>
</tbody>
</table>

Table 1. Performer’s recorded interpretations in this study.

These performances varied greatly in their tempi and interpretative approaches. Half of the interpreters opted to perform the binary repeats with the most original interpretation being that of Fadini who interpolated repeated notes based on the final chord in an ascending fashion at the end of the first half. She also had great variety of rhythmic and dynamic expression, taking clear advantage of the fortepiano’s capabilities. For a proper comparison of the recordings the section repeats and extra audio material, such as Browning’s announcement of his encore, were discarded (Figure 1). Some of the editions varied, with an interesting variation found in ms. 15 in an inner voice, as played by Horowitz, Pletnev, Li, and Lipatti (Musical example 6), which gave continuity to the line G#-A#-B-C#-B begun in ms. 12 (musical example 3).

Differences in Recorded Performances

The audio editor Audacity 1.3.3 was used to look closely at each performance. The following results point to the individual differences in interpretation. As might be expected, the harpsichord revealed less dynamic contrast than the others.

The longest performances were those by Horowitz and Asperen (191"), while the shortest was by Gilels (152") with a close second by Lipatti and Li (156"). The spread difference is 39 seconds (Figure 1).

The slowest initial tempo was by Coleman (86 M.M.) closely followed by Li (87 M.M.) and Horowitz (88 M.M.). The fastest beginning tempo were by Smullyan (97-99 M.M.), who varied slightly, and Fadini (95 M.M.). It was noted that Smullyan performed on a piano whose timbre is thin and light, which could account for his quicker speed. The spread
The difference from fastest to slowest initial tempo is 13 M. M. (Table 2).

In general, the tempi relationships are not exact, given the various discrepancies found from bar to bar within all performers, with the exception of Lipatti who shows remarkably steady rhythmic control and clear phrase intent (Table 2).

### Table 2. Real time Performances (longest to shortest) with repetitions omitted; converted and rounded up to seconds.

<table>
<thead>
<tr>
<th>Performer</th>
<th>Initial Tempo (quarter note)</th>
<th>Duration without repeats</th>
</tr>
</thead>
<tbody>
<tr>
<td>Horowitz</td>
<td>± 98</td>
<td>2'10&quot;9 (2'11&quot;) = 191&quot;</td>
</tr>
<tr>
<td>Asperen</td>
<td>± 92</td>
<td>2'10&quot;9 (3'11&quot;) = 191&quot;</td>
</tr>
<tr>
<td>Smulyan</td>
<td>± 97-99</td>
<td>2'58&quot;2 = 178&quot;</td>
</tr>
<tr>
<td>Browning</td>
<td>± 90</td>
<td>2'49&quot;9 (2'30&quot;) = 170</td>
</tr>
<tr>
<td>Pletnev</td>
<td>± 99</td>
<td>2'49&quot;9 (2'30&quot;) = 170</td>
</tr>
<tr>
<td>Coleman</td>
<td>± 95</td>
<td>2'44&quot;3 = 164&quot;</td>
</tr>
<tr>
<td>Fadini</td>
<td>± 95</td>
<td>2'41&quot;0 = 161&quot;</td>
</tr>
<tr>
<td>Lévi</td>
<td>± 97</td>
<td>2'36&quot;0 = 156&quot;</td>
</tr>
<tr>
<td>Lipatti</td>
<td>± 93</td>
<td>2'35&quot;5 (2'36&quot;) = 156&quot;</td>
</tr>
<tr>
<td>Giesls</td>
<td>± 92</td>
<td>2'31&quot;5 (2'22&quot;) = 152</td>
</tr>
</tbody>
</table>

**Crux occurrences and timings**

Before addressing any interpretative questions or preferences, a measurement of the crux occurrence in each half of the binary sonata was made. Timings of the first half of the sonata by each performer compared with the overall duration without repetitions were also noted. Differences in performances, such as in editorial considerations, were observed and will be duly discussed (Table 3 & Figure 2).
Interpreting the crux in real time. Since the crux occurs in both halves of this sonata followed by a deceptive cadence and a repeat of the entire section (bolero), some musical results might be expected. These are dynamic contrasts and shadings (except in the harpsichord) and/or rubato or agogic elements, such as accelerandi, ritardandi, and so on.

In the first half crux treatment, we found the following:

- Horowitz begins softly and makes a crescendo to ms. 23, then diminuendo to the crux in ms. 25, followed by another crescendo in the crux ms. 25.
- Asperen does not seem to differentiate dynamically, nor agogically. He does use a different registration in the repeat of the first half.
- Smullyan does differentiate dynamically very much. He makes a slight accent on the ascending scalar line before the crux and leaves expressive elements for other places. He interpolates (different edition?) extras bars, which has no bearing on crux expression. These bars are at the end of each half. He repeats ms. 34-37, then ms. 34-35 before taking ms. 38-40 down an octave.
- Browning makes a crescendo beginning in ms. 22 and diminuendo at the pre-crux in ms. 24, finishing the crux at the same dynamic level.
• Pletnev is very expressive, making an accelerando in ms. 22 and ritardando at the pre-crus in ms. 24. He begins ms. 22 with piano crescendo and approaches the crux with great lyricism in cantabile style.

• Coleman makes a slight ritardando in ms. 24 and a crescendo to ms. 25.

• Fadini has a large luftpause before ms. 22 in second repeat with a subito forte. She makes a diminuendo to the crux. Because of the exaggerated dynamic contrasts, Fadini also exaggerates the rhythm and flow of the phrases.

• Li follows the melodic contour, making a crescendo from ms. 22 to ms. 23, then diminuendo to the crux with a slight crescendo-diminuendo again in the crux ms.

• Lipatti is very expressive making both a crescendo and diminuendo according to the melodic contour, starting piano in ms. 22 and diminuendo at the crux bar in ms. Ms. 25.

• Gilels makes an accelerando from ms. 22 with crescendo to the end of ms. 23, followed by diminuendo and normalization of tempo to the crux.

• Pletnev is again very expressive, contrasting lyrical with rhythmic sections. He treats the approach to the crux in the second half much as he did in the first half.

• Coleman makes little expression, keeping phrasing constant.

• Fadini uses great rhythmic variety to express the crux. In the second repeat she makes a large agogic rubato at the end of the crux.

• Li is very expressive with dynamics and rhythmic variety, but less so than Pletnev. He follows much of the same scheme as in the first half but is slightly louder in the repeat of the second half.

• Lipatti is very expressive dynamically following the same scheme as in the first half with a crescendo to the longest note in ms. 61 and following with a diminuendo to prepare the crux in ms. 63.

• Gilels uses both an accelerando and dynamic expression of piano-crescendo from ms. 60 and diminuendo at the end of ms. 61 as in the first half, but to a lesser degree.

Equalized Time Performance

In order to get a more accurate look at the performers’ crux interpretation and its relation to musical structure, the recordings went through a modification in their length - they were time equalized - so as to match a reference absolute time. The software Peak 5 was used for this procedure with repetitions being removed. The process of time equalization was performed so that one could mathematically compare the behavior, in time, of the occurrence of the crux in the equalized recordings in relation to the crux in the reference model. Pletnev’s interpretation with its median length and balanced timing and expression was chosen to be that model. The reference length is therefore 2'49"7. The nine recordings were modified with either a compression or an expansion in length to conform to Pletnev’s median model (Figures 3 & 4; Table 4) with the reference crux occurring at 53".8 and 2'15".8 seconds in parts A and B respectively.
The process to obtain a meaningful comparison is as follows: The ratio between the length of each original recording and Pletnev’s reference length was measured in order to quantify their overall time/length difference. For the recordings that were longer than Pletnev’s (Horowitz, Asperen, and Smullyan) and that had to be compressed, the ratio is represented by a number smaller than one. Likewise, the recordings shorter than Pletnev’s (Chase, Fadini, Li, Lipattu and Gilels) that were time expanded, are given by ratios bigger than the unity.

The overall length adjustments were realized using “Time Equalization Ratios” which were then compared to a new set of ratios - the “Crux Ratios”. These are obtained by dividing the time where the crux occurs in the equalized recordings by the time of the crux in the reference model. By comparing these two different types of ratios (for each of the recordings), one can determine if the crux of the equalized recordings are approximating or deviating to/from the time where the crux of the reference model occur. If the “Crux Ratios” are smaller than the corresponding “Time Equalization Ratios”, then the performances are converging in time around the crux of the model. That convergence indicates that the performers perceive absolute time through the occurrence of meaningful structural materials, i.e. the crux.

We use the following terms and abbreviations to represent these relationships. For example:

1) Time Equalization Ratio (TER) = Time of reference model (TRM) / Time of real time recording (TRTR) or $\text{TER} = \frac{\text{TRM}}{\text{TRTR}}$;

2) Crux Ratio (CR) = Crux of reference model (CRM) / Crux of time-equalized recording (CER) or $\text{CR} = \frac{\text{CRM}}{\text{CER}}$;

3) Section A Ratio (AR) = Time of Section A of equalized recording (AER)/Time of Section A in reference model (ARM) or $\text{AR} = \frac{\text{AER}}{\text{ARM}}$.

Thus, if $\text{CR} < \text{TER} = >$ Crux converges to the reference model’s Crux.

We take as an example the time-equalized recording of Li (Table 4). The data readings are:

a) $\text{TER} = 170 / 156 = 1.08$;
b) $\text{CR section A} = 53.8/56.1 = 0.959 = 1.00$;
c) $\text{CR section B} = 135.8 / 135.6 = 1.001 = 1.00$;
d) $\text{AR} = 87.9 / 89.9 = 0.977 = 1.00$;

Figure 3. Equalized time comparisons in wave form.
Therefore, we find:

e) $1.08 - 1.00 = 0.08$ (total deviation from the real time recording – file was expanded by 8%);

f) $1.00 - 0.959 = 0.04$ (distance from Crux A of model = 4%);

g) $1.00 - 1.001 = 0.001$ (distance from Crux B of model = one tenth of 1%);

h) $1.00 - 1.00 = 0.0$ (distance from end of section A of model = 0%).

Therefore, it follows that because $0.04 < 0.08$, $0.00 < 0.08$, and $0.0 < 0.08$, then convergence is proven.

The following table presents the equalized time-crus relationships to the model.

<table>
<thead>
<tr>
<th>Performer</th>
<th>Reference time/Real Time</th>
<th>Crux A</th>
<th>Total A</th>
<th>Crux B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pletnev</td>
<td>1.00</td>
<td>53.9 = 1.00</td>
<td>87.9 = 1.00</td>
<td>135.8 = 1.00</td>
</tr>
<tr>
<td>Horowitz</td>
<td>0.99</td>
<td>53.9/52.1 = 1.03</td>
<td>87.9/84.8 = 1.04</td>
<td>135.8/130.9 = 1.04</td>
</tr>
<tr>
<td>Asperen</td>
<td>0.99</td>
<td>53.9/52.0 = 1.06</td>
<td>87.9/84.8 = 1.04</td>
<td>135.8/131.4 = 1.03</td>
</tr>
<tr>
<td>Smullyan</td>
<td>0.96</td>
<td>53.9/60.3 = 1.16</td>
<td>87.9/87.2 = 1.01</td>
<td>135.8/125.4 = 1.08</td>
</tr>
<tr>
<td>Browning</td>
<td>1.00</td>
<td>53.9/52.2 = 1.01</td>
<td>87.9/87.2 = 1.01</td>
<td>135.8/133.2 = 1.02</td>
</tr>
<tr>
<td>Coleman</td>
<td>1.03</td>
<td>53.9/53.8 = 1.00</td>
<td>87.9/88.0 = 1.00</td>
<td>135.8/133.8 = 1.02</td>
</tr>
<tr>
<td>Fadini</td>
<td>1.05</td>
<td>53.9/50.1 = 1.07</td>
<td>87.9/87.6 = 1.05</td>
<td>135.8/132.0 = 1.03</td>
</tr>
<tr>
<td>Li</td>
<td>1.08</td>
<td>53.9/56.1 = 1.00</td>
<td>87.9/89.9 = 1.00</td>
<td>135.8/135.5 = 1.00</td>
</tr>
<tr>
<td>Lipatti</td>
<td>1.08</td>
<td>53.9/49.8 = 1.08</td>
<td>87.9/90.1 = 1.00</td>
<td>135.8/134.0 = 1.01</td>
</tr>
<tr>
<td>Gilels</td>
<td>1.11</td>
<td>53.9/50.9 = 1.05</td>
<td>87.9/84.8 = 1.04</td>
<td>135.8/131.9 = 1.03</td>
</tr>
</tbody>
</table>

Table 4. Crux relationships and observations (equalized time).

Analyses of the results (Table 4 & Figure 4) show that 6 out of the 9 (67%) time-equalized recordings converge their crux A towards the model’s crux (all but Smullyan, Fadini and Browning). It also reveals that 7 out of 9 (78%) time-equalized recordings converge their crux B towards the model’s crux (all but Smullyan and Browning) and 8 out of 9 (all except Browning) of the time-equalized recordings converge the end of section A towards the model’s end of A (89%). We consider these findings to be significant, even allowing for a small margin of error.

The “time-equalized” performances keep intact the *agogic* relationships inherent in each of the performances, but they reveal a clear tendency for the crux to converge towards the crux’s time of the reference model. The data gathered strongly points to the existence of two different clocks or two different time dimensions which are simultaneously at play and that relate to the notion of Surface Tempo and Background Tempo, the latter with strong affinities with absolute time.

The Surface Tempo is the temporal dimension where the *agogic* liberties and the creativity of the performers take place. It is kept unchanged during the equalization process, unmoved by modifying the recordings to fit the reference length. Background Tempo, on the other hand, is a deeper temporal dimension that performers intuitively sense through the inner workings of musical structure, such as the crux. At this crucial point of multi-layered structural stability and continuity, performances converge towards absolute time despite their overall Surface Temporal differences.

The procedure described above demands a closer scrutiny of the relationship between interpretation and musical structure at the crux in bi-partite sonata works of Scarlatti.

![Figure 4. Equalized time of Crux A, First half (A), Crux B, and total duration relationships.](image)

Table 4. Crux relationships and observations (equalized time).

**Figure 4.** Equalized time of Crux A, First half (A), Crux B, and total duration relationships.

**Conclusions**

From our study we conclude that there is a tendency for the crux to converge or align in absolute time during the performances. This
implies that there is a general concept of tempo-structure in place amongst most performers.

In the real time performances of Pletnev, Browning, Coleman and Li, despite a considerable (26") variation span in the overall performance times, the placement of crux A is meaningfully convergent. It almost coincides for all four interpretations. When the recordings are made to share the same absolute time, it was further shown that the crux behaves as an *attractor*. Six out of the nine compared recordings were again converging at this singular structural point in the composition.

Of all the interpretations studied, the most expressive was that of Pletnev’s, who, at times, seemed to be the composer, taking additional liberties by adding internal bass notes at certain points. Fadini’s interpretation was viewed as the most original, pulling from the music daring rhythmic and dynamic figures *apropos* of that particular instrument. Lipatti’s sterling and steady interpretation is readily discernible where the performer exhibits their *agogic* choices and differences. Since the long recordings were not the slowest or the shortest the fastest, it can be concluded that the *agogic* variations within Surface Tempo play a fundamental role in the overall differentiation in artistic interpretation. Background Tempo is a deeper more fundamental notion of tempo that links and aligns absolute time with musical structure. Coleman, Smullyan, and Asperen reveal less sensitivity to Background Tempo than Pletnev, Horowitz, Li, Lipatti, Gilels, Browning, and Fadini. The mathematical reasoning in the paper proves that there is a clear tendency of the crux to gravitate and converge towards a place in absolute time, marked by a "time-structure" pointer. This marker is a consequence of the *crux-phi* proportion. The idea that musical structure influences the overall performance in the temporal dimension, which is often intangible, is thus proven.

**References**


http://audacity.sourceforge.net
http://www.bias-inc.com

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ii Rafael Puyana in Suthcliffe (2003).


iv Fadini (2002).

v Kirkpatrick (1953).